A Mathematical Recipe For Success

by Amy Wheeler

Gröbner Bases are like the mathematical version of a recipe.

When your high school math teacher told you that you needed to study because all those variables and equations would turn out to be very practical, you probably didn't realize that they could be used to save your home, catch a flight on time, detect social trends, and follow financial markets. Indeed, Associate Professor Dr. Serkan Hosten even uses complex mathematics to decide the best way possible to get a fire engine to your house.

Hosten's main interest is computing Gröbner Bases. Gröbner Bases are like the mathematical version of a recipe. If an average person were told to cook their favorite restaurant dish, they wouldn't even know where to start. But if the person were able to read a detailed recipe which walked them through the process step by step, the solution would be much easier. That is what Gröbner Bases do for mathematicians working with detailed equations called polynomials; they help the mathematician solve the simple steps so that the answer almost appears right in front of them.

Hosten, a math professor at San Francisco State University (SFSU), San Francisco, California, began his exploration of math a long way from the Golden State. Hosten earned his B.A. in Turkey, which prepared him for his next destination, Cornell University, Ithaca, New York. "I did not have that much fun in my classes in Turkey, but it did prepare me very well for my Ph.D.," says Hosten of his undergraduate experience. He usually studied on Friday nights, and doesn't remember going to too many bars, something he envies in his students, who all seem to be having more fun than he did.

Hosten was always interested in math, but started off his Cornell education concentrating in operations research, which draws on the ideas of engineering and scientific methods to make decisions. While in his first year at Cornell, Hosten met a student working on her mathematics thesis and was impressed with her advisor, Dr. Bernd Sturmfels. Hosten enrolled in one of his classes. "From that point on, after my third semester, I spent a lot more time in the math department. I took a lot more math courses than operations research courses," says Hosten. While his degree is in operations research, he stresses mathematics when conducting research,

especially Gröbner Bases.

Using his knowledge of Gröbner Bases, Hosten can help solve polynomial equations. Polynomial equations appear frequently when describing problems in engineering, transportation, and the economy. An example of a polynomial equation is $7x^4 + 5xy^3 - 9x^2y = 0$, where x and y represent unknown values such as number of people or planes. To continue the recipe idea, these equations might be your favorite dish before you pop it in the oven. The dish is almost complete and only needs to be heated up and it will be ready to serve. The heating up is when Hosten uses the Gröbner Bases to solve for the unknown variables. The equations that Hosten helps to solve are much longer, sometimes containing hundreds of variables. Gröbner Bases are used to solve many of these equations at one time. By solving these equations, Hosten can manipulate all of the variables and find values that will produce the most profitable or most economical outcome.

"For two hundred years mathematicians solved polynomial equations, small ones not big ones. Since they didn't have the tools for actually solving [big polynomial equations] they didn't think about it very hard. So for the last 15 years there has been an explosion," Hosten says. Computers were the technology mathematicians needed to expand this type of research. To solve the multiple equations that he works with, Hosten uses a powerful computer cluster the Mathematics department bought with a grant they received this past summer. The cluster consists of twelve fast processors all chained together. Without this cluster, Hosten says it would be impossible to compute the lengthy equations that he is now working with.

Hosten uses the problems an airline faces to highlight the problems that can be solved with Gröbner Bases and polynomial equations. Suppose a company has a certain flight schedule everyday and they need to assign their flight crews in a cost efficient way. If these were the airline's only stipulations the solution would be easy to find. There would only be a few variables to consider: cost, number of crew, and flight schedule. But airlines also worry about union contracts, which restrict pilots from flying for more than nine hours straight. Airline companies need to get their crews back to the city they are based in, and they prefer that crewmembers won't fly as passengers on a plane but work on their flight back home. And these are just a few of the variables airlines care about. So this type of problem can get very complicated very quickly. An additional complication is that the solution needs to be a whole number, since they cannot have a crew with 2 and one-half flight attendants.

Solving these types of problems involves multiple polynomial equations, which may require mathematicians like Hosten, and Gröbner Bases. Hosten's approach differs from many other mathematicians working with Gröbner Bases. Where others estimate their solutions from non-whole number solutions, Hosten's method makes sure his solutions are always whole numbers. Such high accuracy can mean saving millions of dollars for large companies.

This method of problem solving has many other possible applications. It can also be used by Wall Street financial wizards to create portfolios for maximum profit, or the transportation industry, which needs very cost-efficient ways of operating. Hosten is working with one of his three current graduate students in finding solutions for the San Francisco Fire Department. Bill Storti, mathematics major, has completed all of his classes and is working with Hosten on his thesis. In addition to being a student, Storti is also a lieutenant in the SFFD's assignment office. Along with Hosten, Storti is using his mathematical skill to find ways to assign fire trucks, 21 of them, to the various firehouses so that the fire department can reach 90% of emergencies in 90 seconds. By using mathematical methods, Hosten and Storti may be able to reach that goal while saving the fire department millions

Serkan Hosten
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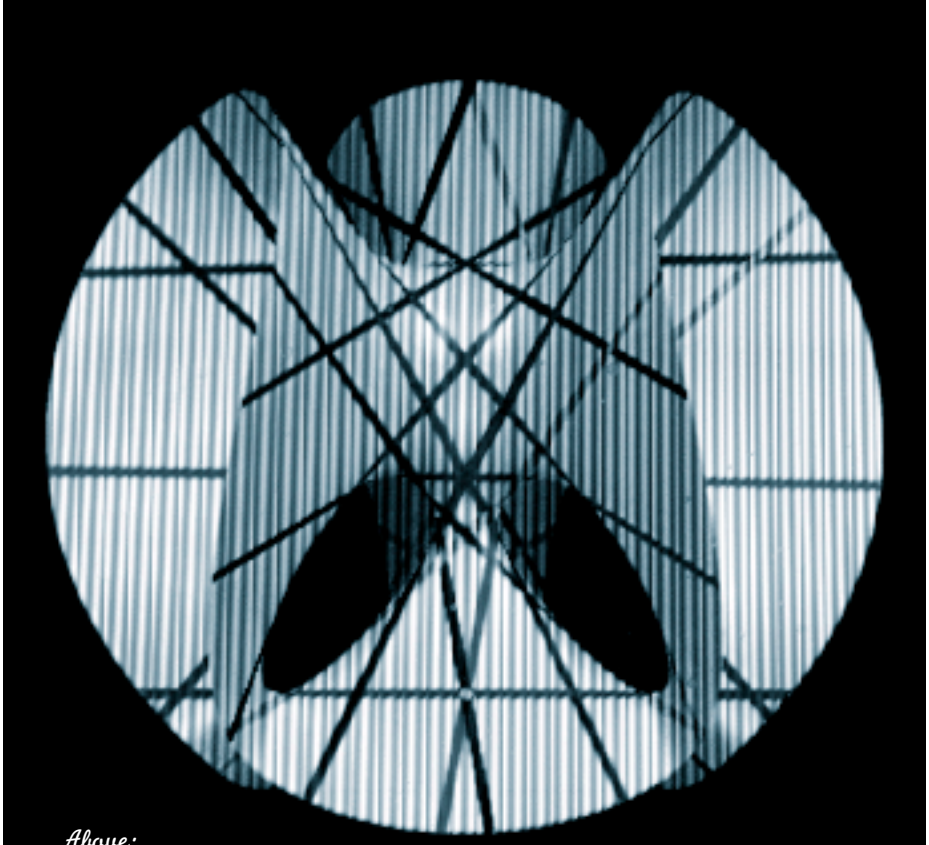
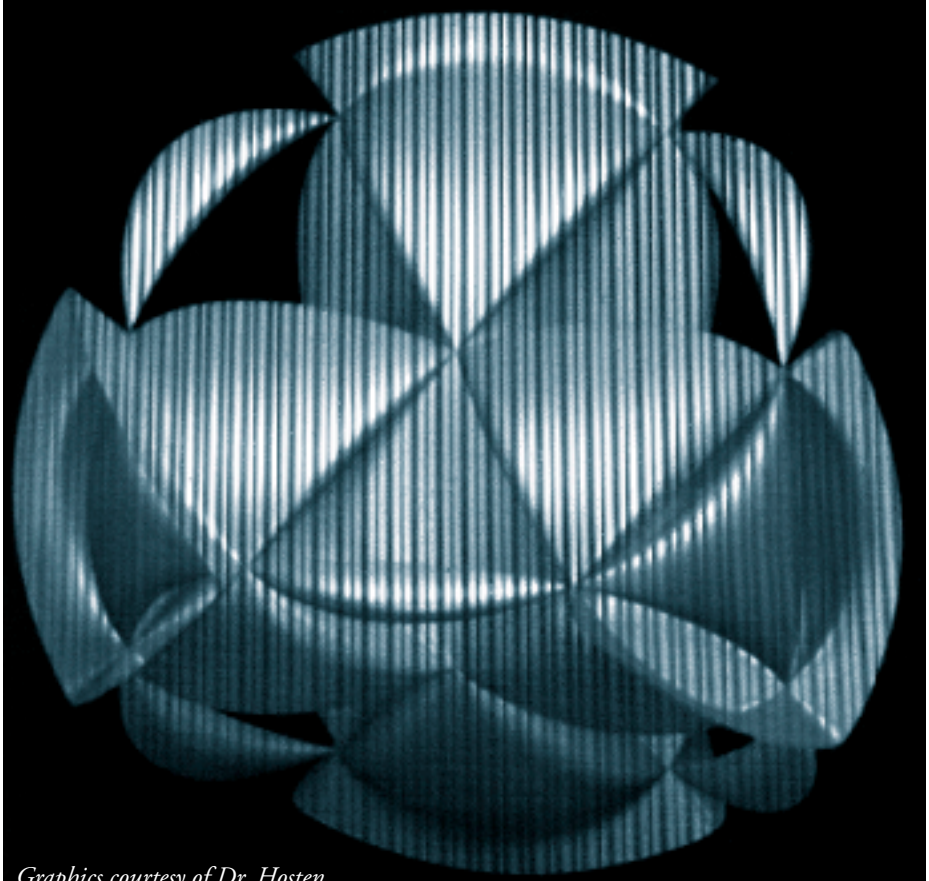


Above:
A cubic surface is an example of objects defined by polynomial equations.

Below:
Gröbner Bases help to find equations for complex geometric shapes.



Graphics courtesy of Dr. Hosten

of dollars that it may have spent buying more fire engines and hiring more fire fighters. "Meeting response time goals is always a challenge, especially with the current budget climate. We are trying to improve the configuration to meet response time goals without any added resources," says Storti.

Hosten continues to look for new areas of math to get involved in. In his future research, Hosten plans to concentrate on a couple of aspects in the broad field of statistics.

For example, if someone were doing research on race, gender, and income levels there would be many different areas they would need to consider for possible trends. Hosten's work will focus on equations that can help someone determine quickly if the number of white, female, middle class is significantly different than the number of Hispanic, female, middle class in a given city. This type of information can be used to help decide if a new store will succeed in a neighborhood or to help a politician decide if he should spend time campaigning in a specific city. Hosten will also be combining his statistical knowledge with biology. Along with a few of his colleagues, Hosten will help construct precise phylogenetic trees. These are lineage trees that chart an animal's ancestors in a branched visual. Building these trees requires statistical analysis of many DNA relationships between distantly-related species. These trees are going to become more and more important, as the human genome is examined in ever finer detail. In a short time, these trees will be used to map disease genes as well as finding minute genetic changes in humans.

So if you meet a young student who is about to give up on their math homework because they say it doesn't have anything to do with real life, consider telling them about Gröbner Bases. And after they ask you what in the world you are talking about, tell them about the firemen, pilots, and stock traders who are depending on mathematics every day to do their jobs better. ✣