This special tetrahedron (a triangular pyramid) was used by J.E. Reeve to exemplify that lattice-point problems, which are at the center of Beck’s research, are much more complicated when stepping from dimension 2 to 3.
Education research shows that most high school students scarcely find math interesting, let alone creative. According to a 2007 study from Public Agenda, a non-profit research organization that reports on public opinion and public policy issues, 76 percent of high school students in Kansas and Missouri see mathematics as “irrelevant” to their lives. This two-state study, tellingly entitled, “Important, but Not for Me,” reflects the results of “national research on public attitudes toward math,” say the authors.

Apparently, math needs a perceptual makeover if students are to see beauty instead of boredom and creativity instead of irrelevance. Perhaps then math could do as Microsoft President Bill Gates suggested in a recent Washington Post op-ed piece, and provide the problem-solving skills students need “to succeed in the knowledge economy.”

Matthias Beck, although on sabbatical from SF State in 2008 at the Mathematical Sciences Research Institute (MSRI) in Berkeley, Beck has agreed to meet in his office to provide insights into the artistic side of mathematics. Beck is a quiet, friendly man with round, wire-rimmed glasses and a quick smile. Originally from Germany, Beck received his Ph.D. from Temple University in Philadelphia, and joined the faculty of the SF State’s Department of Mathematics in 2004. He had, at that time, just finished a stint at the prestigious Max Planck Institute of Mathematics in Bonn, Germany, and an earlier fellowship at MSRI.
Beck’s office is furnished with little more than a small desk, a laptop, a single bookcase, a few houseplants, and a white board covered in symbols and graphs. The view from his ninth floor window stretches west to the Pacific, glittering in sun, but Beck pays little attention to it, being immersed, he says, in an even more captivating landscape.

“I joke about this a little bit,” says Beck. “Sometimes at parties I introduce myself as an artist. But there’s some truth in that joke. There are a lot of parallels between coming up with a theorem or proof and composing a piece of music or painting a picture. It’s very creative. It builds on other people’s work” in much the same way as “musical pieces have evolved from what has gone before, and art, too.”

Hints of Beck’s own creativity lie scattered on the solitary bookshelf. In one section sit cubes, pyramids, and many-pointed stars made of finely polished wood or interlocking plastic pieces. On another rests a set of dice from the game “Dungeons and Dragons.” Shapes are Beck’s passion, he admits. But these particular shapes represent abstractions called “polytopes.” Polytope is a word coined by Alicia Boole, daughter of George Boole, the framer of Boolean logic. It’s a general term that extends the polygon of two dimensions and the polyhedron of three dimensions to any many-faced, sharp-edged, pointy object projecting in any number of dimensions.

“My training,” Beck explains, “is really number theory. I’m sort of on the intersection of number theory-combinatorics. Number theory-combinatorics-geometry.” He laughs. “These fields intersect somewhere, and I’m right there.” Beck, in other words, has carved out a niche for himself in the theoretical realm where geometry and numbers meet.

“Polytopes are the simplest objects you can come up with,” he explains. “They’re objects that are bounded by straight things—straight lines, straight planes. Or in higher dimensions, straight hyperplanes, I guess.” Beck defines polytopes using linear equations such as \( y = 5x + 3 \), or \( x < 2y - 7 \), that determine their sizes and shapes.

Beck fills his many-dimensional polytopes with many-dimensional grids and counts the points, called lattice points, where the grid lines intersect. This gives him a way to determine the volume inside. Even with Beck’s seemingly simple method, however, computing volume is complicated and intuitively, Beck’s method shouldn’t even work.

While grid points are discrete and countable, a volume is a continuous thing. Think of the way water fills a cup. It pours in a continuous stream, not in chunks. As Beck explains it, calculus can determine volume by filling a shape with tiny pieces, sometimes called differential elements. But what makes calculus work is that differential elements...
are not just tiny, they are infinitely so. They are continuous.

“With polyhedra, I don’t have to go to the limit,” says Beck. “I can stay in the discrete world and just apply methods of combinatorics, and get the exact volume. That’s the cool thing.”

Beck’s enthusiasm is infectious and his explanations reveal math’s intellectual appeal. But art inspires us on levels beyond the intellectual, and this leaves a mathematician like Beck to seem, at times, like a painter describing a landscape to a blind person. What does an oak tree look like? The color green? Golden afternoon sunlight? And what about polytopes and other mathematical constructs could illuminate the human soul the way art does?

Such issues are subjective, but even in ancient times, math and art have been joined in the human imagination. As the Greek dramatist Euripides put it, “Mighty are numbers; joined with art, resistless.”

The German astronomer and mathematician Johannes Kepler, who discovered in the 17th century that planetary orbits are ellipses, made a stab at quantifying the math/art relationship. “Geometry,” wrote Kepler, “is the archetype of the beauty of the world.”

One can describe an artful shape with math and an intricate equation with art. From Beck’s simple polytopes to the intricate, dazzling fractal patterns popular on t-shirts and posters, math underlies the images that flood our aesthetic landscape. Math determines perspective. Math denotes pleasing proportions. And math underlies music, from harmonics to fractional beats, time signatures—even the acoustics of the concert hall.

The more one searches for evidence of a math/art connection, the more obvious it becomes. There are peer-reviewed journals devoted to math and art, such as the Journal of Mathematics and the Arts. There are conferences devoted to math and art, such as the yearly Bridges Conference, focusing on “Mathematical Connections in Art, Music, and Science.” There are more math/art websites than Google can track, including the enigmatic Math-Art.net, “where mathematics and art blend into a Zen-like state of peace.”

Of course, there are the artists and musicians known for their mathematical connections, as well. From Leonardo Da Vinci and Johann Sebastian Bach to M. C. Escher and Herbie Hancock, mathematics has informed and transformed works of art. With the advent of computerized music and art, the link has only grown stronger.

For both math and art, the product and the process are intertwined, as so many have discovered at the keyboard, easel, and pottery wheel. The process of mathematical creation, however, is both murkier and more rarified.

Matthias Beck currently does most of his creative math at MSRI, set high in the Berkeley hills above the Lawrence Hall of Science like a big brown polytope with windows. For him, the place is equal parts inspiration and camaraderie. “We’ve got a great space,” says Beck. “There’s always coffee, there’s a library that’s always open, there’s plenty of people to talk to.”
DILATIONS OF A HEPTAGON.

TWO VERY DIFFERENT TRIANGULATIONS OF A CUBE: THE CUBE ON THE LEFT IS DISSECTED INTO SIX TETRAHEDRA, WHILE THE CUBE ON THE RIGHT IS DECOMPOSED INTO 5 TETRAHEDRA. THIS IS IN CONTRAST TO TRIANGULATIONS OF 2-DIMENSIONAL POLYGONS: IF WE TRY TO DECOMPOSE A POLYGON INTO TRIANGLES (WITHOUT INTRODUCING NEW VERTICES), WE WILL ALWAYS END UP WITH THE SAME NUMBER OF TRIANGLES.

THIS PICTURE IS A GEOMETRIC ATTEMPT OF SOLVING THE “FROBENIUS COIN-EXCHANGE PROBLEM”: IN THIS INSTANCE, IMAGINE A CURRENCY SYSTEM WITH 2-, 5-, AND 20- CENT COINS. HOW MANY WAYS ARE THERE TO CHANGE A DOLLAR?
Having other mathematicians to bounce ideas off of is important, since according to Beck, math is a “team sport.” Yet ultimately, Beck conducts his experiments mostly in his own head. “It’s exciting, in part, because there are so few tools necessary. Just you and your brain and maybe a piece of paper.” He pauses. “Paper, a pencil, and a big wastebasket.”

“When I start working on something I usually start with some examples. I don’t exactly design experiments like a physical scientist. But once I go beyond the examples, my work is way more theoretical, and in that sense it’s way more creative.” For Beck, this translates into fun. “It has a lot of playfulness,” he says. “I’m having a blast.”

Mathematical proofs can be a different story. “When you try to prove something interesting in mathematics, it’s very dry in some sense, because you have to be very precise…very clean.” Beck suspects he uses a different part of his brain for proofs than for experiments.

One currently accepted view of the brain, albeit vastly simplified, is that the left hemisphere houses logic and language and is good at analysis and abstraction. Neurologists tend to see the right hemisphere, on the other hand, as the seat of visual, spatial, perceptual, and intuitive information and activity. By this system, the right side is where we do art, and the left side is where we do math. Right?

Not according to Michael W. O’Boyle of Texas Tech University. In a 2005 paper on the brain characteristics of mathematically-gifted teenagers, O’Boyle reports “enhanced processing reliance on the right cerebral hemisphere.” In the young math whizzes, it seems, math is on the art side of the brain.

What science describes, Beck feels. “The first time as a Ph.D. student I proved something that nobody else had thought of before—that was kind of a cool feeling. I think there’s a beauty in mathematics that’s very similar to the beauty that people experience in music or other art forms. So why,” he wonders,” is it not being appreciated?”

Perhaps it is because, for Beck, the product and process of math are also entwined with purpose. As an undergraduate, Beck supported himself, in part, by playing and teaching music. As a graduate student, however, “Math replaced the music for me,” he says, “I think because of the creativity it has. Once you start doing research in mathematics, it’s extremely creative. So there was a time when I stopped playing serious music. Stopped playing constantly. And now I don’t think that’s an accident.”

For Matthias Beck, mathematics has become a medium of creation, a means of connecting with and expressing to the world around him. In a word, his art.

Math is a medium just waiting for others to discover, says Beck—both as artists and as audience members—and Beck wants to help. In addition to teaching university students, Beck is involved in area Math Circles, where math teachers and scholars introduce middle-school and high-school kids to math beyond simple calculation. He hopes that once they see math’s creative possibilities, at least some will find their muse. In the bargain, they may also learn the problem-solving skills they need to “succeed in the knowledge economy.”

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