

Business Calculus

(Techniques of Differentiation)



Campus Academic Resource Program

[Techniques of Differentiation]

Instead of using the derivative notation f' , another notation $\frac{dy}{dx}$ is used often and it means “the derivative with respect to x .”

Power Rule

$$\begin{aligned} f(x) = x^n &\rightarrow f'(x) = nx^{(n-1)} & f(x) = x^5 &\rightarrow f'(x) = 5x^4 & f(x) = x^{(-4)} &\rightarrow f'(x) = -4x^{(-5)} \\ f(x) = cx^n &\rightarrow f'(x) = cnx^{(n-1)} & f(x) = 8x^4 &\rightarrow f'(x) = 8 \cdot 4x^3 = 32x^3 \\ f(x) = c &\rightarrow f'(x) = 0 & f(x) = 122 &\rightarrow f'(x) = 0 \end{aligned}$$

(The derivative of a constant number is 0)

Product Rule

$$\frac{d}{dx} [f(x)g(x)] = f'(x)g(x) + f(x)g'(x) \qquad \frac{d}{dx} [(x^3-4)(x^3+x^8)] = (3x^2-0)(x^3+x^8) + (x^3-4)(3x^2+8x^7)$$

Quotient Rule

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2} \qquad \frac{d}{dx} \left[\frac{x^4}{(x^4-4)} \right] = \frac{4x^3(x^4-4) - x^4(x^4-4)}{[(x^4-4)]^2}$$

Chain Rule

$$\frac{d}{dx} [f(u)] = f'(u) \frac{du}{dx} \qquad \text{(The } u \text{ is a differentiable function of } x \text{, then the composite } f(u) \text{ is a differentiable function of } x \text{.)}$$

[Example] $\frac{d}{dx} [(x^3 - x^4)^{50}]$

Let $u = x^3 - x^4$ and $f(u) = u^{50}$, then $f'(u) = 50u^{49}$ and $\frac{du}{dx} = 3x^2 - 4x^3$

We can solve this by following the instruction above.

$$\frac{d}{dx} [(x^3 - x^4)^{50}] = 50u^{49} (3x^2 - 4x^3) = 50(x^3 - x^4)^{49} (3x^2 - 4x^3)$$

Derivative of the Natural Logarithm

$$\frac{d}{dx} [\ln(x)] = \frac{1}{x} \qquad \frac{d}{dx} [3\ln(x)] = 3 \cdot \frac{1}{x} = \frac{3}{x}$$

Derivative of the Logarithm with Base b

$$\frac{d}{dx} [\log_b x] = \frac{1}{x \ln b} \qquad \frac{d}{dx} [\log_5 x^4] = \frac{d}{dx} [4 \log_5 x] = \frac{4}{x \ln 5}$$

Derivative of e^x

$$\frac{d}{dx} [e^x] = e^x \qquad \frac{d}{dx} [4e^x] = 4e^x$$