

# Business Calculus

*(Linear Models, Quadratic Function, and Exponential Function)*

**Linear Models** – The use of linear functions to show relationships in the real world

## [Key Terms]

**Total Cost** -- Variable cost + Fixed cost

**Revenue** -- The total payment received by business and usually calculated by multiplying the unit price by the number of units sold

**Profit** -- Calculated by subtracting the total cost  $C(x)$  from revenue

**Break Even Point** -- The number of units or the amount of dollars at which break-even occurs and revenue  $R(x)$  = total cost  $C(x)$  at the break-even

## [Cost Linear Function]

$$C(x) = mx + b$$

**$mx$  (variable cost)** -- expenses that change proportionally to the output level (ex. raw material, energy consumption, hourly wage, etc.)

**$b$  (fixed cost)** -- expenses that do not depend on the output level (ex. rent, fixed salary, etc.)

**$m$  (slope and marginal cost)** – cost that measures the incremental cost per additional one unit

## [Example]

The Tacos Heaven is a new fast food restaurant in San Francisco. The Tacos Heaven sold 1,100 tacos in June and the total cost was \$1,810. In July, the restaurant sold 2,000 tacos and the total cost was \$2,800.

a) Find the marginal cost and the monthly fixed cost, and establish a linear cost function based on the information. (Hint: slope of a linear function = marginal cost)

b) The Tacos Heaven sells tacos for \$2.00 each. Find the revenue and the profit for the June and the July.

c) How many tacos must the Tacos Heaven sell in a month to obtain break-even?

**Quadratic Function** – The simplest function with a graph that is not a straight line

A quadratic function can be written in a form

$$f(x) = ax^2 + bx + c \quad \text{or} \quad y = ax^2 + bx + c$$

where  $a, b,$  and  $c$  are fixed number. ( $a \neq 0$ )

(Example)  $f(x) = 4x^2 - 2x + 6$  ( $a=4, b=-2,$  and  $c=6$ )

## [Key Terms]

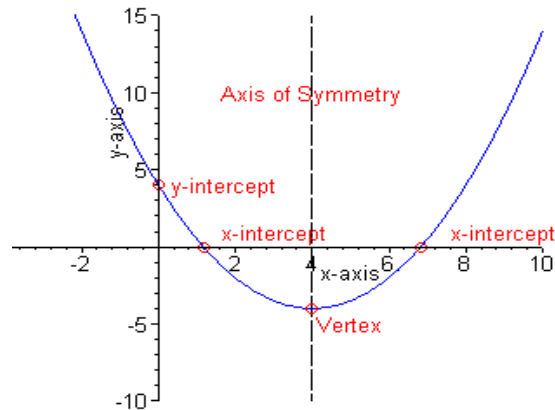
**x-Intercept** – Point(s) where a graph crosses x-axis and found when  $f(x)=0$

The x value(s) at x-intercept(s) can be found with the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**y-Intercept** – Point where graph crosses y-axis and found when  $x = 0$

**Vertex** – The turning point of the parabola where x-coordinate is  $\frac{-b}{2a}$  and y-coordinate is  $f(\frac{-b}{2a})$



(Source: <http://ppl.lonestar.edu/~bmagan/ParabolasPuttingItAllTogether/>)

## Exponential Function

[The general laws of exponents]

1.  $x^a x^b = x^{a+b}$       2.  $\frac{x^a}{x^b} = x^{a-b}$       3.  $\frac{1}{x^a} = x^{-a}$       4.  $x^0 = 0$  ( $x \neq 0$ )

5.  $(x^a)^b = x^{ab}$       6.  $(xy)^a = x^a y^a$       7.  $(\frac{x}{y})^a = \frac{x^a}{y^a}$

[Two Key Terms]

**Exponential growth** – A rate of growth that becomes faster as the amount of the thing that is growing

A function experiences exponential growth if it has the form  $y = Ab^t$  where  $b > 1$

**Compound interest** – Interest that is calculated on both the money that was lent or borrowed and the interest already earned or charged

[Compound Interest]

$A(t) = P(1 + r/m)^{mt}$  A - Future Value P - Present value

t - The number of years r - interest rate