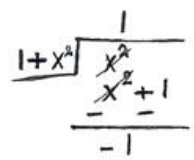
Calculus Worksheet Quiz Answers

1. $Let\ u = arc\ tanx\ and\ dv = 2xdx \Rightarrow du = \dfrac{1}{1+x^2}dx\ and\ v = x^2$

$$\int (2x \cdot arctanx)dx = x^2\ arc\ tanx - \int x^2 \frac{1}{1+x^2}dx$$

Doing long division simplifies the problem. The long division is performed below:



$$\Rightarrow 1 - \frac{1}{1+x^2}$$

$$= x^2\ arc\ tanx - \int \left(1 - \frac{1}{1+x^2}\right)dx = x^2\ arc\ tanx - \int 1\ dx + \int \frac{1}{1+x^2}dx$$

$$= x^2\ arc\ tanx - x + arc\ tanx + C$$

2. $\int_0^2 (5x+3)dx$

$$\Delta x_i = \frac{2-0}{n} = \frac{2}{n}\ for\ i = 1,2,3, \ldots .n$$

$$Choose\ the\ sampling\ points, c_i = 0 + \left(\frac{2-0}{n}\right)i = \frac{2i}{n}\ for\ i = 1,2,3, \ldots n$$

$$f(x) = 5x + 3$$

$$\int_0^2 (5x+3)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\Delta x_i = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(\frac{2i}{n}\right)\left(\frac{2}{n}\right) = \lim_{n\to\infty}\sum_{i=1}^{n}\left(5\cdot\frac{2i}{n}+3\right)\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{20i}{n^2}+\frac{6}{n}\right) = \lim_{n\to\infty}\left\{\sum_{i=1}^{n}\left(\frac{20i}{n^2}\right)+\sum_{i=1}^{n}\left(\frac{6}{n}\right)\right\} = \lim_{n\to\infty}\left\{\frac{20}{n^2}\sum_{i=1}^{n} i + n\cdot\frac{6}{n}\right\}$$

$$Formulas: \sum_{i=1}^{n}(1) = n \; and \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$= \lim_{n\to\infty}\left\{\frac{20}{n^2}\cdot\frac{n(n+1)}{2}+6\right\} = \lim_{n\to\infty}\left\{\frac{10n^2+5n}{n^2}+6\right\} = \lim_{n\to\infty}\left\{\frac{10n^2}{n^2}+\frac{5n}{n^2}+6\right\}$$

$$= \lim_{n\to\infty}\left\{10+\frac{5}{n}+6\right\} = 16+0 = 16$$

3. *Let r be the radius of the cylinder and h be the height.*

The total volume of the cylinder, $V = \pi r^2 h = 25\pi$.

Therefore, $h = \frac{25\pi}{r^2\pi} = \frac{25}{r^2}$

Cost, C = *Cost per top surface area + Cost per bottom surface area*

+Cost per curved surface area

Total Surface area of a cylinder = top surface area + bottom surface area + curved surface area

$$= \pi r^2 + \pi r^2 + 2\pi rh$$

$$= \$5\cdot\pi r^2 + \$5\cdot\pi r^2 + \$4(2\pi rh) = \$(10.\pi r^2 + 8\pi rh)$$

Substituting the value of h, in the above equation:

$$C = 10\pi r^2 + 8\pi r\cdot\frac{25}{r^2} = 10\pi r^2 + \frac{200\pi}{r}$$

$$C(r) = 10\pi r^2 + \frac{200\pi}{r}$$

$C'(r) = 20\pi r + 200\pi\left(-\frac{1}{r^2}\right) = 20\pi r - \frac{200\pi}{r^2} = \frac{20\pi r^3 - 200\pi}{r^2} = \frac{20\pi(r^3 - 10)}{r^2} = 0$

$20\pi(r^3 - 10) = 0$

$r^3 - 10 = 0$

$r = \sqrt[3]{10}m = 2.15m$

$h = 5.41m$

$C = \$437.46$

4. $\int \frac{x^2 + x - 1}{x(x^2 - 1)} dx = \int \frac{x^2 + x - 1}{x(x+1)(x-1)} dx = \int (\frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}) dx$

Dealing with the above partial fractions:

$A(x+1)(x-1) + Bx(x-1) + Cx(x+1) = x^2 + x - 1$

It is quite convenient to solve using different (somewhat arbitrary) values of x.

$x = 0 \Longrightarrow -A = -1 \Longrightarrow A = 1$

$x = 1 \Longrightarrow 2C = 1 \Longrightarrow C = \frac{1}{2}$

$x = -1 \Longrightarrow 2B = -1 \Longrightarrow B = -\frac{1}{2}$

$= \int \left(\frac{1}{x} - \frac{1}{2(x+1)} + \frac{1}{2(x-1)}\right) dx = \ln|x| - \frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + C$

$= \ln|x| - \frac{1}{2} ln \frac{|x+1|}{|x-1|} + C$

5. $\int \sqrt{1+\sqrt{5+\sqrt{x}}}\,dx$

$(Remove\ outside\ squares\ first)$

$Let\ 1+\sqrt{5+\sqrt{x}} = u^2$

$u = \sqrt{1+\sqrt{5+\sqrt{x}}}$

$\Longrightarrow \sqrt{5+\sqrt{x}} = u^2 - 1$

$\Longrightarrow \sqrt{x} = (u^2-1)^2 - 5$

$\Longrightarrow x = ((u^2-1)^2-5)^2$

$$\frac{dx}{du} = \frac{d((u^2-1)^2-5)^2}{d((u^2-1)^2-5)} \cdot \frac{d((u^2-1)^2-5)}{d(u^2-1)} \cdot \frac{d(u^2-1)}{du}$$

$$= 2((u^2-1)^2-5)\cdot 2(u^2-1).\,2u$$

$$dx = 2[(u^4-2u^2+1-5)\cdot(4u^3-4u)]du$$

$$= 8[u^7-3u^5-2u^3+4u]du$$

$$= (8u^7-24u^5-16u^3+32u)du$$

$$\int \sqrt{1+\sqrt{5+\sqrt{x}}}\,dx = \int u\cdot(8u^7-24u^5-16u^3+32u)du$$

$$= \int (8u^8-24u^6-16u^4+32u^2)du$$

$$= \tfrac{8}{9}u^9 - \tfrac{24}{7}u^7 - \tfrac{16}{5}u^5 + \tfrac{32}{3}u^3 + C$$

$$= \frac{8}{9}(\sqrt{1+\sqrt{5+\sqrt{x}}})^9 - \frac{24}{7}(\sqrt{1+\sqrt{5+\sqrt{x}}})^7 - \frac{16}{5}(\sqrt{1+\sqrt{5+\sqrt{x}}})^5 + \frac{32}{3}(\sqrt{1+\sqrt{5+\sqrt{x}}})^3 + C$$

$$= \frac{8}{9}\left(1+\sqrt{5+\sqrt{x}}\right)^{\frac{9}{2}} - \frac{24}{7}\left(1+\sqrt{5+\sqrt{x}}\right)^{\frac{7}{2}} - \frac{16}{5}\left(1+\sqrt{5+\sqrt{x}}\right)^{\frac{5}{2}} + \frac{32}{3}\left(1+\sqrt{5+\sqrt{x}}\right)^{\frac{3}{2}} + C$$

$6. Differentiate:$

$i. y = e^{6x^2+5x-8}$

$Let\ u = 6x^2 + 5x - 8. Chain\ Rule: \dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$

$\frac{dy}{du} = e^{6x^2+5x-8}$ $\frac{du}{dx} = \frac{d}{dx}(6x^2 + 5x - 8) = 12x + 5$

$\Longrightarrow \frac{dy}{dx} = (12x + 5)e^{6x^2+5x-8}$

$ii. y = 5\ sin^2(x^3)$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$$

$Let\ u = sin(x^3) \Longrightarrow \frac{dy}{du} = \frac{d(u^2)}{du} = 2u = 2\ sin(x^3)$

$Let\ v = x^3 \Longrightarrow \dfrac{du}{dv} = \dfrac{d(\sin v)}{dv} = \cos(v) = cos(x^3)$

$\dfrac{dv}{dx} = 3x^2$

$$\Longrightarrow \frac{dy}{dx} = 5 \cdot 2\,\mathrm{Sin}(x^3)\, Cos(x^3) \cdot 3x^2$$

$$= 15x^2\big(2\,\mathrm{Sin}(x^3)\, Cos(x^3)\big)$$

$$= 30x^2\,\mathrm{Sin}(x^3)\, Cos(x^3)$$

$The\ given\ expression\ can\ be\ simplified\ further\ in\ the\ following\ way$:

$2\,\mathrm{Sin}(x^3)\,Cos(x^3)\ can\ also\ be\ simplified\ to\ \ \mathrm{Sin}(2x^3)\ \ using\ the\ trigonometric\ identity$

$Sin2A = 2SinA \cdot CosA$

$Therefore, 30x^2\,\mathrm{Sin}(x^3)\,Cos(x^3) = 15x^2\big(2\,\mathrm{Sin}(x^3)\,Cos(x^3)\big) = 15\,\mathrm{Sin}(2x^3)$