Campus Academic Resource Program

Business Calculus

(Techniques of Differentiation)

[Techniques of Differentiation]

Instead of using the derivative notation f', another notation $\frac{dy}{dx}$ is used often and it means "the derivative with respect to x."

Power Rule

$f(x) = x^n \longrightarrow f'(x) = nx^{(n-1)}$ $f(x) = x^5 \longrightarrow f'(x) = 5x^4$ $f(x) = x^{(-4)} \longrightarrow f'(x) = -4x^{(-5)}$

$f(x) = cx^n \longrightarrow f'(x) = cnx^{(n-1)}$ $f(x) = 8x^4 \longrightarrow f'(x) = 8 \cdot 4x^3 = 32x^3$

$f(x) = c \longrightarrow f'(x) = 0$ $f(x) = 122 \longrightarrow f'(x) = 0$ (The derivative of a constant number is 0)

Product Rule

$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$ $\frac{d}{dx}[(x^3-4)(x^3+x^8)] = (3x^2-0)(x^3+x^8)+(x^3-4)(3x^2+8x^7)$

Quotient Rule

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$ $\frac{d}{dx}\left[\frac{x^4}{(x^4-4)}\right] = \frac{4x^3(x^4-4) - x^4(x^4-4)}{[(x^4-4)]^2}$

Chain Rule

$\frac{d}{dx}[f(u)] = f'(u)\frac{du}{dx}$ **(The u is a differentiable function of x, then the composite f(u) is a differentiable function of x.)**

[Example] $\frac{d}{dx}[(x^3 - x^4)^{50}]$

Let $u = x^3 - x^4$ and $f(u) = u^{50}$, then $f'(u) = 50u^{49}$ and $\frac{du}{dx} = 3x^2 - 4x^3$

We can solve this by following the instruction above.

$\frac{d}{dx}[(x^3 - x^4)^{50}] = 50u^{49}(3x^2 - 4x^3) = 50(x^3 - x^4)^{49}(3x^2 - 4x^3)$

Derivative of the Natural Logarithm

$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$ $\frac{d}{dx}[3\ln(x)] = 3 \cdot \frac{1}{x} = \frac{3}{x}$

Derivative of the Logarithm with Base b

$\frac{d}{dx}[\log_b x] = \frac{1}{x\ln b}$ $\frac{d}{dx}[\log_5 x^4] = \frac{d}{dx}[4\log_5 x] = \frac{4}{x\ln 5}$

Derivative of e^x

$\frac{d}{dx}[e^x] = e^x$ $\frac{d}{dx}[4e^x] = 4e^x$