Business Calculus

(Logarithms, Limits, Average Rate of Change, Instantaneous Rate of Change)

[Logarithm]

In logarithm, an exponential function $b^y = x$ is written,

$\log_b x = y$,where b is the base and y is the power.

For example, an exponential form $2^4 = 16$ is written in logarithm,

$$\log_2 4 = 16$$

Logarithm Identities

$\log_b xy = \log_b x + \log_b y$ $\quad$ $\log_b \frac{x}{y} = \log_b x - \log_b y$

$\log_b x^r = r \cdot \log_b x$ $\quad$ $\log_b b = 1 \quad \log_b 1 = 0$

$\log_b \frac{1}{x} = -\log_b x$ $\quad$ $\log_b x = \frac{\log_a x}{\log_a b}$

$\log_b b^x = x$ $\quad$ $b^{\log_b x} = x$

Exponential Decay Model $\quad$ $Q(t) = Q_0e^{-kt}$

Exponential Growth Model $\quad$ $Q(t) = Q_0e^{kt}$

$\mathbf{Q_0}$ represents the value of Q at time t = 0 (initial value), and k is the decay or growth constant.

[Limits]

$$\lim_{x \to a}[f(X)] = L$$

If f(x) gets close to the number L as x **approaches** *a* from both negative (left) and positive (right), then the limit of f(x) is L as x approaches *a*.

Example: $\lim_{x \to 3} f(x) = 5$ (the limit of f(x) equals to 5 as x approaches 3)

Limit Laws

$\lim_{x\to a}[f(x) + g(x)] = \lim_{x\to a}[f(x)] + \lim_{x\to a}[g(x)]$

$\lim_{x\to a}[f(x) - g(x)] = \lim_{x\to a}[f(x)] - \lim_{x\to a}[g(x)]$

$\lim_{x\to a}[c\, f(x)] = c \cdot \lim_{x\to a}[f(x)]$

$\lim_{x\to a}[f(x)\, g(x)] = \lim_{x\to a}[f(x)] \cdot \lim_{x\to a}[g(x)]$

$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a}[f(x)]}{\lim_{x\to a}[g(x)]}$ $[\ \lim_{x\to a}[g(x)] \neq 0\ ]$

[Average Rate of Change]

Average rate of change of S = $\frac{\text{Change in S}}{\text{Change in t}} = \frac{\Delta S}{\Delta t} = \frac{S(b) - S(a)}{b-a}$

Example: The following shows the gas price per gallon for three months. Calculate the average rate of change of the gas price between January and March.
January-$2.20
February-$2.50
March-$2.80

Average rate of change of the gas price = $\frac{\$2.80-\$2.20}{3(\text{March})-1(\text{January})}$ = 0.3 $ / month

[Instantaneous Rate of Change]

The instantaneous rate of change is found by using a limit

$$\lim_{h\to 0}\frac{f(a+h) - f(a)}{h}$$

Also, the limit expression above gives instantaneous velocity at time *t*.

$$V = \lim_{h\to 0}\frac{s(t+h) - s(t)}{h}$$

Derivative Function

If f is a function, its derivative function f' (f prime) is the function whose value f'(x) is the derivative of f at x. f' associates to each x the **slope** of the tangent to the graph of the function f at x, or the rate of change of f at x. The formula for the derivative function is

$$f'(x) = \lim_{h\to 0}\frac{f(a+h) - f(a)}{h}$$