

Calculus Worksheet Quiz Answers

1. Let $u = \arctan x$ and $dv = 2x dx$ $du = \frac{1}{1+x^2} dx$ and $v = x^2$

$$\int (2x \cdot \arctan x) dx = x^2 \arctan x - \int x^2 \frac{1}{1+x^2} dx$$

Doing long division simplifies the problem. The long division is performed below:

$$\begin{array}{r} 1 \\ 1+x^2 \overline{) x^2} \\ \underline{x^2} \\ -1 \end{array}$$

$$1 - \frac{1}{1+x^2}$$

$$= x^2 \arctan x - \int \left(1 - \frac{1}{1+x^2}\right) dx = x^2 \arctan x - \int 1 dx + \int \frac{1}{1+x^2} dx$$

$$= x^2 \arctan x - x + \arctan x + C$$

2. $\int_0^2 (5x + 3) dx$

$$\Delta x_i = \frac{2-0}{n} = \frac{2}{n} \text{ for } i = 1, 2, 3, \dots, n$$

Choose the sampling points, $c_i = 0 + \left(\frac{2-0}{n}\right)i = \frac{2i}{n}$ for $i = 1, 2, 3, \dots, n$

$$f(x) = 5x + 3$$

$$\int_0^2 (5x + 3)dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i)\Delta x_i = \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(\frac{2i}{n}\right)\left(\frac{2}{n}\right) = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(5 \cdot \frac{2i}{n} + 3\right)\left(\frac{2}{n}\right)$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{20i}{n^2} + \frac{6}{n}\right) = \lim_{n \rightarrow \infty} \left\{ \sum_{i=1}^n \left(\frac{20i}{n^2}\right) + \sum_{i=1}^n \left(\frac{6}{n}\right) \right\} = \lim_{n \rightarrow \infty} \left\{ \frac{20}{n^2} \sum_{i=1}^n i + n \cdot \frac{6}{n} \right\}$$

Formulas: $\sum_{i=1}^n (1) = n$ and $\sum_{i=1}^n i = \frac{n(n+1)}{2}$

$$= \lim_{n \rightarrow \infty} \left\{ \frac{20}{n^2} \cdot \frac{n(n+1)}{2} + 6 \right\} = \lim_{n \rightarrow \infty} \left\{ \frac{10n^2 + 5n}{n^2} + 6 \right\} = \lim_{n \rightarrow \infty} \left\{ \frac{10n^2}{n^2} + \frac{5n}{n^2} + 6 \right\}$$

$$= \lim_{n \rightarrow \infty} \left\{ 10 + \frac{5}{n} + 6 \right\} = 16 + 0 = 16$$

3. Let r be the radius of the cylinder and h be the height.

The total volume of the cylinder, $V = \pi r^2 h = 25\pi$.

Therefore, $h = \frac{25\pi}{r^2\pi} = \frac{25}{r^2}$

Cost, $C = \text{Cost per top surface area} + \text{Cost per bottom surface area}$

+Cost per curved surface area

Total Surface area of a cylinder = top surface area + bottom surface area + curved surface area

$$= \pi r^2 + \pi r^2 + 2\pi r h$$

$$= \$5 \cdot \pi r^2 + \$5 \cdot \pi r^2 + \$4(2\pi r h) = \$(10 \cdot \pi r^2 + 8\pi r h)$$

Substituting the value of h , in the above equation:

$$C = 10\pi r^2 + 8\pi r \cdot \frac{25}{r^2} = 10\pi r^2 + \frac{200\pi}{r}$$

$$C(r) = 10\pi r^2 + \frac{200\pi}{r}$$

$$C'(r) = 20\pi r + 200\pi \left(-\frac{1}{r^2}\right) = 20\pi r - \frac{200\pi}{r^2} = \frac{20\pi r^3 - 200\pi}{r^2} = \frac{20\pi(r^3 - 10)}{r^2} = 0$$

$$20\pi(r^3 - 10) = 0$$

$$r^3 - 10 = 0$$

$$r = \sqrt[3]{10}m = 2.15m$$

$$h = 5.41m$$

$$C = \$437.46$$

$$4. \int \frac{x^2 + x - 1}{x(x^2 - 1)} dx = \int \frac{x^2 + x - 1}{x(x+1)(x-1)} dx = \int \left(\frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}\right) dx$$

Dealing with the above partial fractions:

$$A(x+1)(x-1) + Bx(x-1) + Cx(x+1) = x^2 + x - 1$$

It is quite convenient to solve using different (somewhat arbitrary) values of x .

$$x = 0 \quad -A = -1 \quad A = 1$$

$$x = 1 \quad 2C = 1C = \frac{1}{2}$$

$$x = -1 \quad 2B = -1B = -\frac{1}{2}$$

$$= \int \left(\frac{1}{x} - \frac{1}{2(x+1)} + \frac{1}{2(x-1)}\right) dx = \ln|x| - \frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + C$$

$$= \ln|x| - \frac{1}{2} \ln \frac{|x+1|}{|x-1|} + C$$

$$5. \int \sqrt{1 + \sqrt{5 + \sqrt{x}}} dx$$

(Remove outside squares first)

$$\text{Let } 1 + \sqrt{5 + \sqrt{x}} = u^2$$

$$u = \sqrt{1 + \sqrt{5 + \sqrt{x}}}$$

$$\square \sqrt{5 + \sqrt{x}} = u^2 - 1$$

$$\square \sqrt{x} = (u^2 - 1)^2 - 5$$

$$\square x = ((u^2 - 1)^2 - 5)^2$$

$$\frac{dx}{du} = \frac{d((u^2 - 1)^2 - 5)^2}{d((u^2 - 1)^2 - 5)} \cdot \frac{d((u^2 - 1)^2 - 5)}{d(u^2 - 1)} \cdot \frac{d(u^2 - 1)}{du}$$

$$= 2((u^2 - 1)^2 - 5) \cdot 2(u^2 - 1) \cdot 2u$$

$$dx = 2[(u^4 - 2u^2 + 1 - 5) \cdot (4u^3 - 4u)] du$$

$$= 8[u^7 - 3u^5 - 2u^3 + 4u] du$$

$$= (8u^7 - 24u^5 - 16u^3 + 32u) du$$

$$\int \sqrt{1 + \sqrt{5 + \sqrt{x}}} dx = \int u \cdot (8u^7 - 24u^5 - 16u^3 + 32u) du$$

$$= \int (8u^8 - 24u^6 - 16u^4 + 32u^2) du$$

$$= \frac{8}{9}u^9 - \frac{24}{7}u^7 - \frac{16}{5}u^5 + \frac{32}{3}u^3 + C$$

$$= \frac{8}{9}(\sqrt{1 + \sqrt{5 + \sqrt{x}}})^9 - \frac{24}{7}(\sqrt{1 + \sqrt{5 + \sqrt{x}}})^7 - \frac{16}{5}(\sqrt{1 + \sqrt{5 + \sqrt{x}}})^5 + \frac{32}{3}(\sqrt{1 + \sqrt{5 + \sqrt{x}}})^3 + C$$

$$= \frac{8}{9} \left(1 + \sqrt{5 + \sqrt{x}}\right)^{\frac{9}{2}} - \frac{24}{7} \left(1 + \sqrt{5 + \sqrt{x}}\right)^{\frac{7}{2}} - \frac{16}{5} \left(1 + \sqrt{5 + \sqrt{x}}\right)^{\frac{5}{2}} + \frac{32}{3} \left(1 + \sqrt{5 + \sqrt{x}}\right)^{\frac{3}{2}} + C$$

6. Differentiate:

i. $y = e^{6x^2+5x-8}$

Let $u = 6x^2 + 5x - 8$. Chain Rule: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$$\frac{dy}{du} = e^{6x^2+5x-8} \quad \frac{du}{dx} = \frac{d}{dx}(6x^2 + 5x - 8) = 12x + 5$$

$$\square \frac{dy}{dx} = (12x + 5)e^{6x^2+5x-8}$$

ii. $y = 5 \sin^2(x^3)$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$$

Let $u = \sin(x^3)$ $\square \frac{dy}{du} = \frac{d(u^2)}{du} = 2u = 2 \sin(x^3)$

Let $v = x^3$ $\frac{du}{dv} = \frac{d(\sin v)}{dv} = \cos(v) = \cos(x^3)$

$$\frac{dv}{dx} = 3x^2$$

$$\begin{aligned} \square \frac{dy}{dx} &= 5 \cdot 2 \sin(x^3) \cos(x^3) \cdot 3x^2 \\ &= 15x^2 (2 \sin(x^3) \cos(x^3)) \\ &= 30x^2 \sin(x^3) \cos(x^3) \end{aligned}$$

The given expression can be simplified further in the following way:

$2 \sin(x^3) \cos(x^3)$ can also be simplified to $\sin(2x^3)$ using the trigonometric identity

$$\sin 2A = 2 \sin A \cdot \cos A$$

Therefore, $30x^2 \sin(x^3) \cos(x^3) = 15x^2(2 \sin(x^3) \cos(x^3)) = 15 \sin(2x^3)$