BUSINESS CALCULUS

Campus Academic Resource Program

QUIZ 3 – DIFFERENTIATION

1. Solve the following *basic* derivatives using the power, constant and logarithm rules:

a) f(x) = $7x^3$ f'(x) = $21x^2$

b) g(x) = $4x^{-3}$ g'(x) = $-12x^{-4}$

c) h(y) = $3y^{4.5} - 2y^{3.3} - 7y^{2.1} + 2y^{-1.8}$ h'(y) = $13.5y^{3.5} - 6.6y^{2.3} - 14.7y^{1.1} - 3.6y^{-2.8}$

d) R(x) = $3e^x$ R'(x) = $3e^x$

e) Q(t) = 5ln(t) Q'(t) = $\frac{5}{t}$

2. Use the product and/or quotient rule as appropriate:

All answers are factored.

a) C(x) = $(3x^3)e^x$ C'(x) = $3x^2e^x(x+3)$

b) f(t) = $\frac{3t+e^t}{2t^3-5}$ f'(t) = $\frac{3+e^t}{2t^3-5} - \frac{6t^2(3t+e^t)}{(2t^3-5)^2}$

3. The next problems involve all of the above rules, and include the chain rule. (hint: sometimes it is easier to apply log rules *first*, then differentiate.)

All answers are factored.

a) g(t) = $(2t^3 - t^2)^3$ g'(t) = $6t^5(2t-1)^2(3t-1)$

b) h(z) = $e^{3z^2} + \frac{z^2\ln(z)}{z-1}$ h'(z) = $6ze^{3z^2} + \frac{z(z+z\ln(z)-2\ln(z)-1)}{(z-1)^2}$

c) f(w) = $\log_2(w^3 - 3w)$ f'(w) = $\frac{3(w+1)(w-1)}{w(w^2-3)\ln(2)}$

d) h(s) = $\ln(e^{s^4-2s^2})$ h'(s) = $4s(s-1)(s+1)$

4. The cost and revenue functions for North Vernon GPS Devices (NV GPS).

R(x) = 160 + 220x – 1.5x^2 , where x is in number of GPS units manufactured

C(x) = 1.2x^2 + 7.2x − 13.3, where x is in number of GPS units manufactured

a) What are the marginal cost, marginal revenue and marginal profit functions for NV GPS?

Marginal cost: C'(x) = 2.4x + 7.2

Marginal revenue: R'(x) = 220-3.0x

Profit: P(x) = R(x) – C(x) = 160 + 220x − 1.5x^2 − 1.2x^2 − 7.2x + 13.3

= -2.7x^2 + 212.8x + 173.3

Marginal Profit: P'(x) = -5.4x + 212.8

b) How many units must be manufactured for the maximum profit?

Since the curve for marginal profit is curved down, the maximum for the function P(x) can be found when P'(x) is set equal to zero:

0 = -5.4x + 212.8

-5.4x = -212.8 therefore x = 39.4, or 40 units manufactured.

Fig 1. Graph of P(x) and P'(x)=0