

Business Calculus

Quiz 2 – Linear Models, Quadratic Functions and Exponential Functions

1) Solve the following exponential functions:

a) $(5x^2y)(2x^3y) = 10x^5y^2$

b) $\frac{3x^3y^4}{27x^{-4}y^7} = \frac{x^7}{3y^3}$

c) $(10q^{-2})^3 = 1000q^{-6}$

d) $\left(\frac{12a^{23.2}b^{34.8}c^{406.2}}{241a^{-15.5}b^{14.7}c^{-227.9}}\right)^0 = 1$

Luna's Books sold 128 books in June. The total cost that month was \$1,700. In October, Luna's Books sold 248 books and the total cost was \$3,500.

2) What is the marginal cost, monthly fixed cost and the linear cost equation for Luna's books?

$(x_1, y_1) = (128, 1700)$, $(x_2, y_2) = (248, 3500)$

$m = \left(\frac{3500-1700}{248-128}\right) = \frac{1800}{120} = 15$

$y = 15(x - 128) + 1700 = 15x - 1920 + 1700 = 15x - 220$

Marginal cost: \$15.00

Monthly fixed cost: \$220.00

Linear cost equation: $y = \$15.00(x) = \220.00

3) The average price for each book at Luna's is \$17.00. What is the revenue and profit for June and October?

June: $R(x) = \$17.00(128) = \2176

October: $R(x) = \$17.00(248) = \4216

June: $P(x) = R(x) - C(x) = 2176 - 15(128) - 220 = \36

October: $P(x) = 4216 - 15(248) - 220 = \276

4) How many books must Luna's sell in order to make a profit of \$5,000?

$$5,000 = 17(x) - 15(x) - 220$$

$$5,000 = 2x - 220$$

$$4780 = 2x$$

$$2390 = x$$

Luna's must sell at least 2390 books

After expanding her business and creating a website, Luna's new cost function is given below:

$$C(x) = \$21,500 + 150x - 25x^2$$

5) What is the vertex (highest point) of the above cost function?

x-point of the vertex: $\frac{b}{2a}$

$$a = -25, b = 150, c = 21500$$

$$-\frac{150}{(2)(-25)} = \frac{150}{50} = 3$$

$$y = 21500 + 150(3) - 25(3^2) = 21500 + 450 - 225 = 21725$$

Vertex is at (3, 21725)

6) What are the x and y intercepts of this cost function?

x – intercepts can be found using the quadratic formula:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\$ \frac{-150 \pm \sqrt{150^2 - (4)(-25)(21500)}}{2(-25)} = \$ \frac{-150 \pm 1473.94}{-50} = \$-26.48 \text{ \& } \$32.48$$

x-intercepts: (-26.48, 0), (32.48, 0)

y-intercept: $21500 + 150(0) - 25(0^2) = \$21,500 \rightarrow (0, 21500)$

7) After starting to turn a profit, Luna’s Books starts a business savings account for future expansion. Her goal is to make \$500,000 to expand into a shop at the local mall in five years. If she invests her money at a rate of 6.5% compounded monthly, what should her initial investment be?

$$P_0 \left(1 + \frac{r}{m}\right)^{mt} = P$$

Where P_0 is the initial investment, m = compounding period, r = rate of interest, t = years of investment and P is the final amount after investment. Therefore we must solve for P_0 :

$$P_0 \left(1 + \frac{0.065}{12}\right)^{(12)(60)} = \$500,000$$

$$P_0 (48.89) = \$500,000$$

$$P_0 = 10,228.04$$