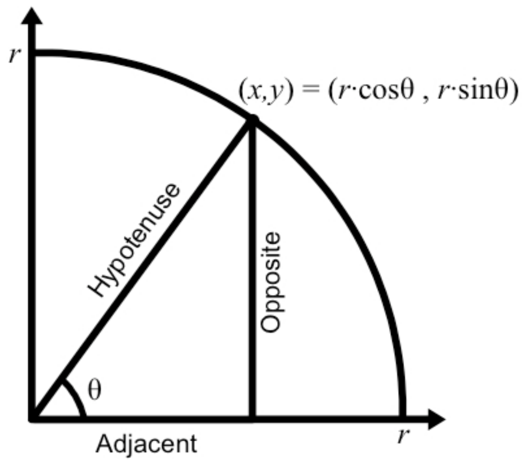
Trigonometry

Campus Academic Resource Program



Definitions

$$\sin(\theta)=\frac{Opp}{Hyp}=\frac{y}{r} \qquad \csc(\theta)=\frac{Hyp}{Opp}=\frac{r}{y}$$

$$\cos(\theta)=\frac{Adj}{Hyp}=\frac{x}{r} \qquad \sec(\theta)=\frac{Hyp}{Adj}=\frac{r}{x}$$

$$\tan(\theta)=\frac{Opp}{Adj}=\frac{y}{x} \qquad \cot(\theta)=\frac{Adj}{Opp}=\frac{x}{y}$$

Inverse Identities

$$\sin(\theta)=\frac{1}{\csc(\theta)} \quad \text{or} \quad \csc(\theta)=\frac{1}{\sin(\theta)}$$

$$\cos(\theta)=\frac{1}{\sec(\theta)} \quad \text{or} \quad \sec(\theta)=\frac{1}{\cos(\theta)}$$

$$\tan(\theta)=\frac{1}{\cot(\theta)}=\frac{\sin(\theta)}{\cos(\theta)}$$

$$\cot(\theta)=\frac{1}{\tan(\theta)}=\frac{\cos(\theta)}{\sin(\theta)}$$

Pythagorean Identities

$$\sin^2(\theta)+\cos^2(\theta)=1$$

$$\tan^2(\theta)+1=\sec^2(\theta)$$

$$\cot^2(\theta)+1=\csc^2(\theta)$$

Sum-Angle Identity

$$\cos(\theta\pm\phi)=\cos(\theta)\cos(\phi)\mp\sin(\theta)\sin(\phi)$$

$$\sin(\theta\pm\phi)=\sin(\theta)\cos(\phi)\pm\sin(\phi)\cos(\theta)$$

$$\tan(\theta\pm\phi)=\frac{\tan(\theta)\pm\tan(\phi)}{1\mp\tan(\theta)\tan(\phi)}$$

Euler's Formula

$$e^{i\theta}=\cos(\theta)+i\cdot\sin(\theta) \quad \textit{where} \quad i^2=-1$$

Converting Radian and Degrees

$$x\text{ rad}=\frac{180x^\circ}{\pi} \quad \text{and} \quad x^\circ=\frac{\pi\cdot x}{180}\text{rad}$$

Double-Angle Identities

$$\begin{aligned}\cos(2\theta) &= \cos^2(\theta)-\sin^2(\theta)\\ &= 2\cos^2(\theta)-1\\ &= 1-2\sin^2(\theta)\end{aligned}$$

$$\sin(2\theta)=2\sin(\theta)\cos(\theta)$$

$$\tan(2\theta)=\frac{2\tan(\theta)}{1-\tan^2(\theta)}$$

Half-angle Identities

$$\sin\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1-\cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1+\cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right)=\frac{1-\cos(\theta)}{\sin(\theta)}$$

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Special Angles

Angle		**sin(*θ*)**	**cos(*θ*)**	**tan(*θ*)**
0°	***0***	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	Undefined

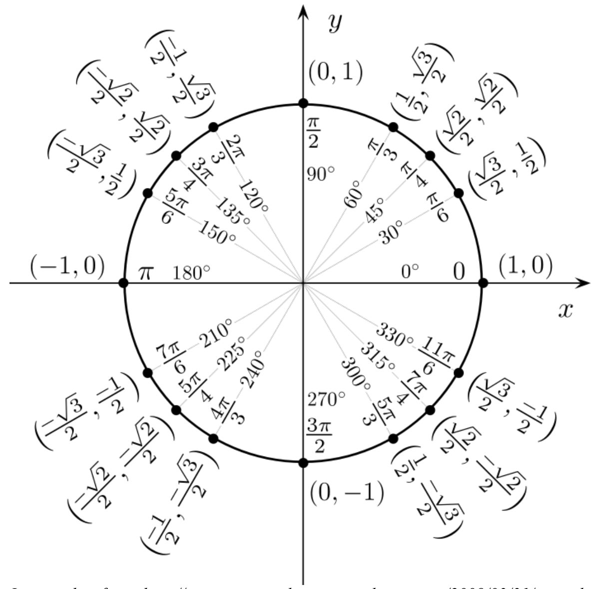

Image taken from: http://necessaryconclusions.wordpress.com/2008/03/31/special-angles/

Inverse Trigonometric Functions

Function	***Domain***	***Range***
$y=\sin^{-1}(x)$	$[-1,1]$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$y=\cos^{-1}(x)$	$[-1,1]$	$[0,\pi]$
$y=\tan^{-1}(x)$	$(-\infty,\infty)$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
$y=\sec^{-1}(x)$	$(-\infty,\infty)$	$(0,\pi)$
$y=\csc^{-1}(x)$	$(-\infty,-1]\cup[1,\infty)$	$\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$
$y=\cot^{-1}(x)$	$(-\infty,-1]\cup[1,\infty)$	$\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$

Law of Sine

$$\frac{\sin(\alpha)}{A}=\frac{\sin(\beta)}{B}=\frac{\sin(\gamma)}{C}$$

Law of Cosine

$$C^2=A^2+B^2-2AB\cdot\cos(\gamma)$$

Area of a Triangle

$$Area=\frac{1}{2}AB\cdot\sin(\gamma)$$