Trigonometry Quiz Solutions

CARP Campus Academic Resource Program

1. First notice that the $\sin(120°)$ is the same as the $\sin(60°)$ in the 2nd quadrant.

(a) $\sin(120°) = \sin(60°) = \frac{\sqrt{3}}{2}$

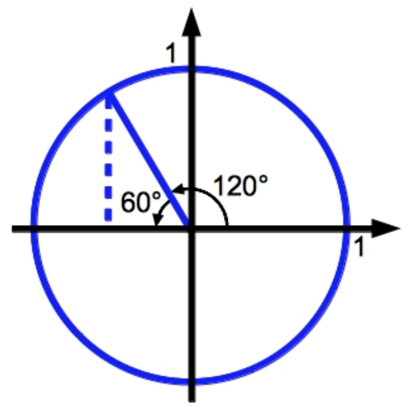
(b) $\cos(120°) = -\cos(60°) = -\frac{1}{2}$

(c) $\tan(120°) = -\tan(60°) = -\sqrt{3}$

(d) $\csc(120°) = \frac{1}{\sin(120°)} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$

(e) $\sec(120°) = \frac{1}{\cos(120°)} = \frac{1}{-\frac{1}{2}} = -2$

(f) $\cot(120°) = \frac{1}{\tan(120°)} = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$



2. First notice that $\frac{13\pi}{4}$ is more than 2π so it goes around the circle once and is the same as $\frac{5\pi}{4}$, so we'll replace $\frac{13\pi}{4}$ with $\frac{5\pi}{4}$. This is also the same as a $\frac{\pi}{4}$ angle measured from the negative *x*-axis.

(a) $\sin\left(\frac{13\pi}{4}\right) = \sin\left(\frac{5\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

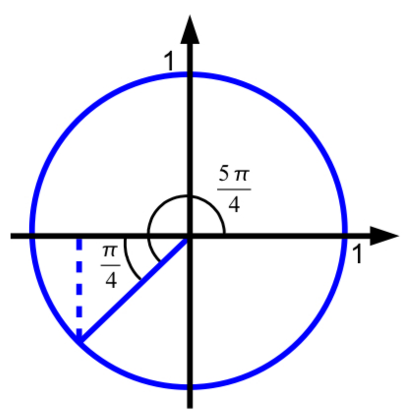
(b) $\cos\left(\frac{13\pi}{4}\right) = \cos\left(\frac{5\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

(c) $\tan\left(\frac{13\pi}{4}\right) = \tan\left(\frac{5\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

(d) $\csc\left(\frac{13\pi}{4}\right) = \frac{1}{\sin\left(\frac{13\pi}{4}\right)} = \frac{1}{-\frac{\sqrt{2}}{2}} = -\sqrt{2}$

(e) $\sec\left(\frac{13\pi}{4}\right) = \frac{1}{\cos\left(\frac{13\pi}{4}\right)} = \frac{1}{-\frac{\sqrt{2}}{2}} = -\sqrt{2}$

(f) $\cot\left(\frac{13\pi}{4}\right) = \frac{1}{\tan\left(\frac{13\pi}{4}\right)} = \frac{1}{1} = 1$



3. $a = 3$ and $c = 5$ so you can calculate b using the Pythagorean theorem: $a^2 + b^2 = c^2$ therefore, $b = \sqrt{c^2 - a^2} = \sqrt{5^2 - 3^2} = 4$.

(a) $\sin(\theta) = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{a}{c} = \dfrac{3}{5}$

(b) $\cos(\theta) = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{b}{c} = \dfrac{4}{5}$

(c) $\tan(\theta) = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{a}{b} = \dfrac{3}{4}$

(d) $\sin(\phi) = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{b}{c} = \dfrac{4}{5} = \cos(\theta)$

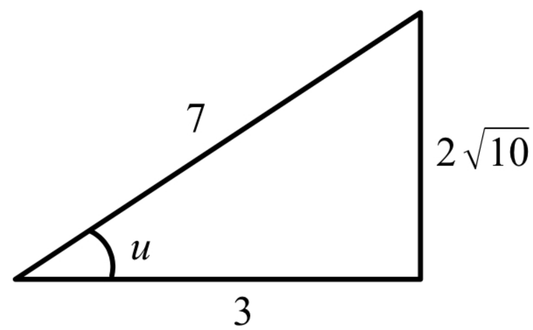
(e) $\cos(\phi) = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{a}{c} = \dfrac{3}{5} = \sin(\theta)$

(f) $\cot(\phi) = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{a}{b} = \dfrac{3}{4} = \tan(\theta)$

4. $\cos(u) = \dfrac{\text{Adj}}{\text{Hyp}} = \dfrac{3}{7}$ So, Adj = 3 and Hyp = 7 By the Pythagorean Theorem you can get Opp = $2\sqrt{10}$.

This gives: (a) $\sin(u) = \dfrac{\text{Opp}}{\text{Hyp}} = \dfrac{2\sqrt{10}}{7}$ and (b)

$\tan(u) = \dfrac{\text{Opp}}{\text{Adj}} = \dfrac{2\sqrt{10}}{3}$



Alternative Solution:

(a) We know $\sin^2\theta + \cos^2\theta = 1$, so

$\sin^2\theta + \left(\dfrac{3}{7}\right)^2 = 1$. Therefore, $\sin u = \pm\sqrt{1 - \dfrac{9}{49}} = \pm\dfrac{2\sqrt{10}}{7}$. Since $0 \le u \le \pi$, sine is always positive.

(b) We know $\sec^2\theta = \tan^2\theta + 1$, which is the same as $\dfrac{1}{\cos^2\theta} = \tan^2\theta + 1$. Therefore,

$\dfrac{1}{\cos^2 u} = \tan^2 u + 1$. Solving for $\tan(u)$ gives $\tan(u) = \pm\dfrac{2\sqrt{10}}{3}$. Again, $0 \le u \le \pi$ so

$\tan(u) = \dfrac{2\sqrt{10}}{3}$.

5. (a) $(\sin\theta + \cos\theta)^2 = \sin^2(\theta) + \cos^2(\theta) + 2\sin(\theta)\cos(\theta)$ Multiply it out.

$= 1 + 2\sin(\theta)\cos(\theta)$ Since $\sin^2(\theta) + \cos^2(\theta) = 1$.

$= 1 + \sin(2\theta)$ By the double angle identity.

(b) $\dfrac{1 - \cos(2\theta)}{2} = \dfrac{1 - (\cos^2\theta - \sin^2\theta)}{2}$ By using the double angle identity for cosine.

$= \dfrac{1 - \cos^2\theta + \sin^2\theta}{2}$ By Pythagorean Thm $\cos^2\theta = 1 - \sin^2\theta$

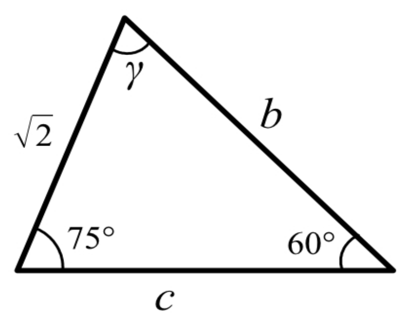
$= \dfrac{1 - (1 - \sin^2\theta) + \sin^2\theta}{2}$ Simplify

$$= \frac{2\sin^2\theta}{2}$$
$$= \sin^2\theta$$

(c) $\cos^2\left(\frac{x}{2}\right) = \left(\pm\sqrt{\frac{1+\cos\theta}{2}}\right)^2$ By the half-angle identity for cosine.

$$= \frac{1+\cos\theta}{2}$$
$$= \frac{1+\cos\theta}{2}\cdot\frac{(1-\cos\theta)}{(1-\cos\theta)}$$
$$= \frac{1-\cos^2\theta}{2-2\cos(\theta)}$$ Recall $1-\cos^2\theta = \sin^2\theta$
$$= \frac{\sin^2\theta}{2-2\cos(\theta)}$$

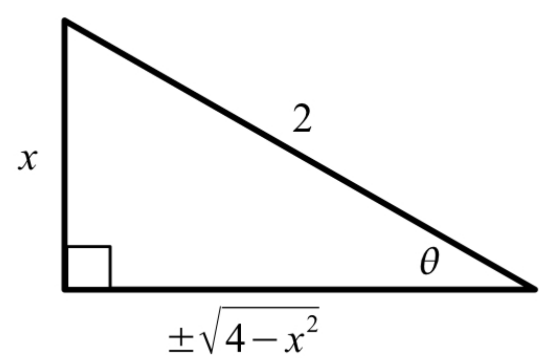
6. γ can be solved for with the face that the sum of angles in a triangle must be $180°$.
$75° + 60° + \gamma = 180°$
$\gamma = 45°$
Next you can use the law of sine to a.



$$\frac{\sin 45°}{\sqrt{2}} = \frac{\sin 60°}{a}$$
$$a = \frac{\sqrt{2}\sin 60°}{\sin 45°} = \frac{\sqrt{2}\frac{\sqrt{3}}{2}}{\frac{\sqrt{2}}{2}} = \sqrt{3}$$

Next you can use law of cosine to solve for b.
$$b^2 = a^2 + c^2 - 2ab\cos(\beta)$$
$$= 2+3-2\sqrt{2}\sqrt{3}\cos(75°) = 5-2\sqrt{6}\cos(45°-30°)$$
$$= 5-2\sqrt{6}(\cos 45°\cos 30° - \sin 45°\sin 30°)$$
$$= 5-2\sqrt{6}\left(\frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2}\right)$$
$$= 2+\sqrt{3} \quad \text{So} \quad b = \sqrt{2+\sqrt{3}}$$

7. Since $y = \tan\left(\sin^{-1}\left(\frac{x}{2}\right)\right)$, let $\theta = \sin^{-1}\left(\frac{x}{2}\right)$.
i.e. $\sin(\theta) = \frac{x}{2} = \frac{Opp}{Hyp}$. So with respect to θ, the opposite side is x and the hypotenuse is 2 which makes the adjacent side equal to $\pm\sqrt{4-x^2}$. Therefore:



$$y = \tan\left(\sin^{-1}\left(\frac{x}{2}\right)\right) = \tan(\theta) = \frac{Opp}{Adj}$$
$$= \pm\frac{x}{\sqrt{4-x^2}}$$