

## BUSINESS CALCULUS

### QUIZ 1: LOGARITHMS, LIMITS, RATES OF CHANGE

1. Solve the following equations:

$$2^x = 64 \quad x = 6$$

$$3^y = \frac{1}{9} \quad y = -2$$

$$15^n = 1 \quad n = 0$$

$$\log_5 x = 3 \quad x = 125$$

$$\log_7 2401 = y \quad y = 4$$

2. Expand the following logarithmic expressions:

$$\log_3 \left( \frac{3x^2}{8y^3} \right)^5 = 5[(\log_3 3 + 2\log_3 x) - (\log_3 8 + 3\log_3 y)]$$

$$\ln(9x^{-3}y^2)^5 = 5[\ln(9) - 3\ln(x) + 2\ln(y)]$$

3. Solve the following equation:

$$\ln\left(\frac{x}{2}\right)^{-2} = -4 \quad x = \underline{\hspace{2cm}}$$

$$x = 2e^2 \text{ and } x = -2e^2$$

4. A student wants to save \$15,000 for graduate school. If the student is starting out with \$7,500, how long will it take to make his goal, at 4.5% interest compounded continuously?

$$\text{Continuous compounding: } P = P_0 e^{rt}$$

Where  $P_0$  = initial investment,  $r$  = rate of interest,  $t$  = time of investment in years,  $P$  is amount returned after  $t$  years.

Therefore:

$$15000 = 7500e^{(0.045*t)} \rightarrow \ln(2) = 0.045*t \rightarrow t = 15.4 \text{ years}$$

5. What is the limit of each equation?

$$\lim_{x \rightarrow 0} \frac{1}{x^4} \quad \text{Limit: } \infty$$

$$\lim_{x \rightarrow \infty} \frac{3x^5 + 7x^2 - 5}{4x^5 - 12x^3 + 3x + 4} \quad \text{Limit: } \frac{3}{4}$$

6. Let  $f(x) = 3x^2$  What is the average rate of change of  $f(x)$  between  $x=2$  and  $x=3$ ?

$$\frac{f(3) - f(2)}{3 - 2} = 27 - 12 = 15$$

What is the instantaneous rate of change of  $f(x)$  at  $x=2$ ?

Instantaneous change:

$$\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{3(2^2 + 2(2)h + h^2) - 3(2^2)}{h} = \lim_{h \rightarrow 0} \frac{12h + 3h^2}{h} = \lim_{h \rightarrow 0} 12 + 3h = 12$$

7. The cost of a monthly MUNI pass in 1980 was \$16.00. In 2010 it was \$60.00. What was the average rate of change in the cost of a MUNI pass?

(1980, 16) and (2010, 60)

$$\frac{60 - 16 \text{ dollars}}{2010 - 1980 \text{ years}} = \frac{44}{30} = \frac{22}{15} = 1.47 \text{ dollars/year}$$

8. The cost equation for the manufacturing of Canjura's Large Complex Widgets is given as follows:

$$C(x) = 5 + 3x - 10x^2 \quad (\text{Where } x \text{ is in units of widgets.})$$

What is the average rate of change between 10 units and 12 units?

$$\frac{f(12) - f(10)}{12 - 10} = \frac{-1399 - (-965)}{2} = -217$$

What is the instantaneous rate of change at 10 units?

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{f(10+h) - f(10)}{h} &= \lim_{h \rightarrow 0} \frac{5 + 3(10) + 3(h) - 10(10)^2 - 20(10)h - 10h^2 - 5 - 3(10) + 10(10)^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{3h - 200h - 10h^2}{h} = \lim_{h \rightarrow 0} 3 - 200 - 10h = -197 \end{aligned}$$